Probabilities in Daily Life
Part 2 - Bayes

www.rationality-freiburg.de
ea-freiburg.org

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Rationality

- The **desire** to see reality as it really is, not as you want it to be.
  - Does not mean you are succeeding
  - Does not mean you will ever succeed
- **Epistemic** rationality: Understanding reality.
- **Instrumental** rationality: Tools and exercises for achieving your goals.

- www.lesswrong.com
- www.rationality-freiburg.de

Effective Altruism (EA)

- You have a moral obligation to do good in the world
- If you want to do good, you should try to be effective and not wasteful (e.g. give your time or money to the right people)

- 80000hours.org
- www.givewell.org
- www.ea-freiburg.org

Recap

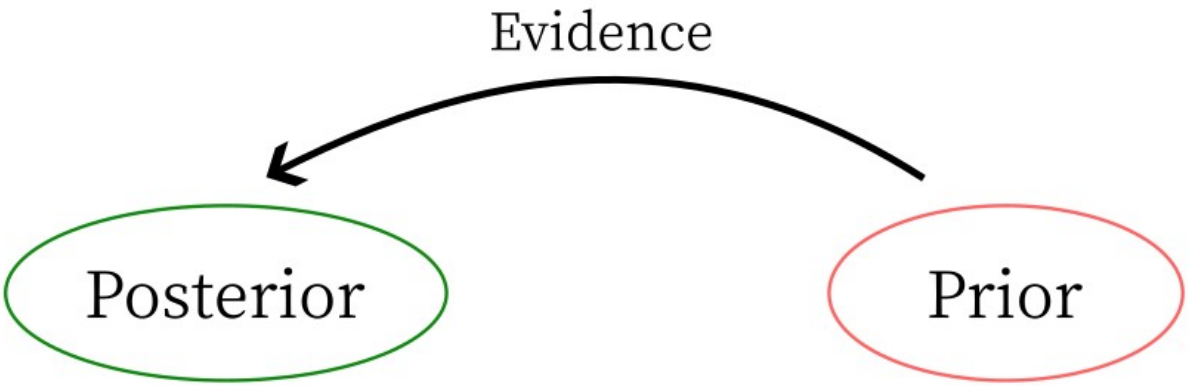
- Event
- Probability
- P(A and B) = P(A) * P(B|A)
- P(A or B) = P(A) + P(B) – P(A and B)
- Why apply them in Daily Life?

Conditional Probability

- P(A) → P(speaks English)
- P(B) → P(speaks German)
- P(A|**B**) → P(speaks English|**speaks German**)
- P(B|**A**) → P(speaks German|**speaks English**)

- Come up with examples
- Some where P(A|B) very similar to P(B|A)
- Some where P(A|B) very different to P(B|A)

Updating



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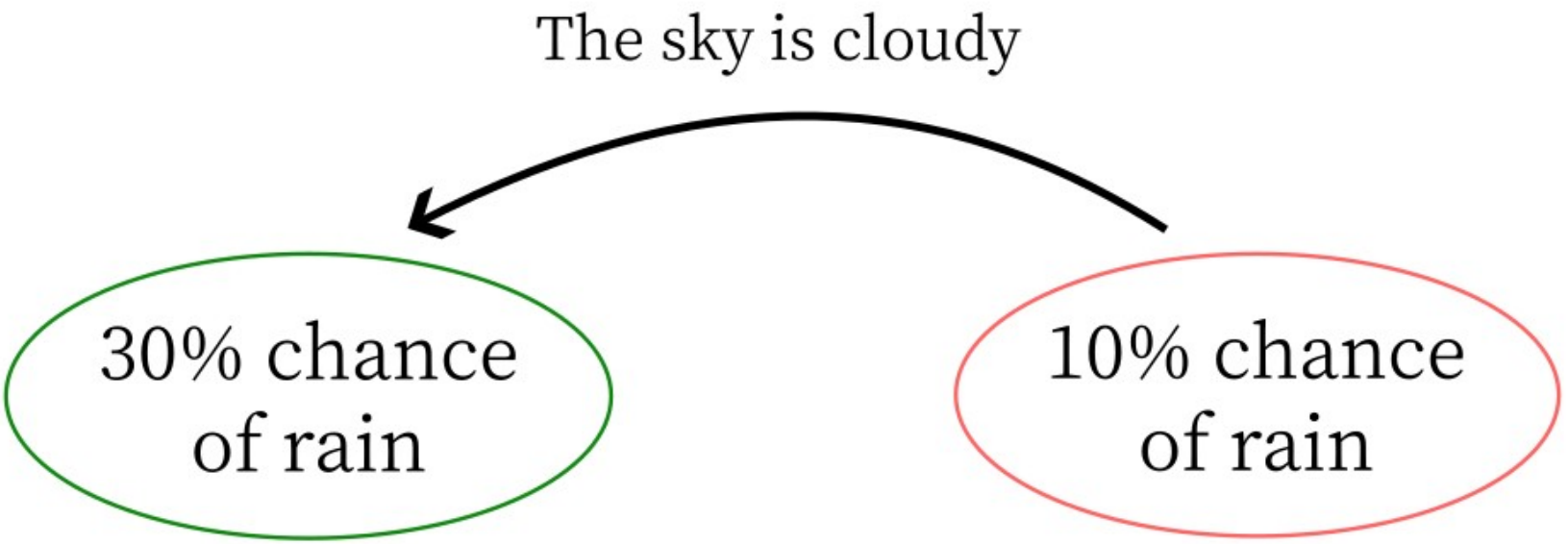

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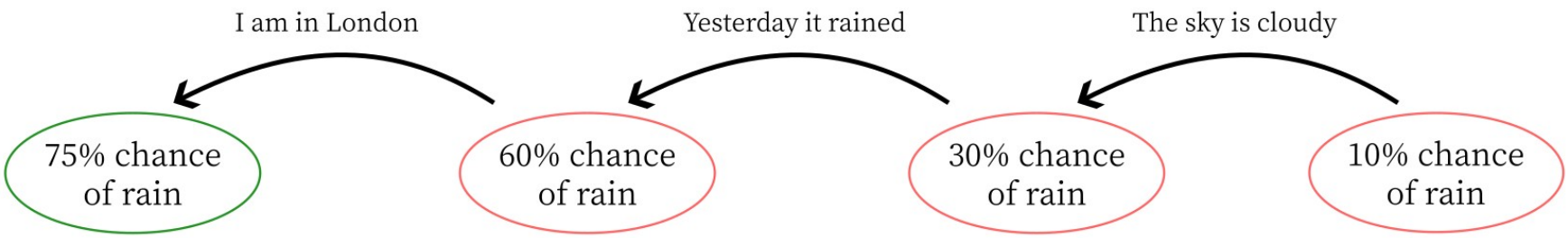

I am in London
Yesterday it rained
The sky is cloudy
75% chance of rain
60% chance of rain
30% chance of rain
10% chance of rain

Bayes‘ Theorem

- Reverend Thomas Bayes (1740s)
- Pierre-Simon Laplace (1774)

Bayes‘ Theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$$P(married|have\ children) = \frac{P(have\ children|married) \cdot P(married)}{P(have\ children)}$$

Bayes‘ Theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\neg A) \cdot P(\neg A)}$$

$$P(married|have\ children) = \frac{P(have\ children|married) \cdot P(married)}{P(have\ children|married) \cdot P(married) + P(have\ children|not\ married) \cdot P(not\ married)}$$

Bayes‘ Theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + P(B|A_3) \cdot P(A_3)}$$

$$P(married|have\ children) = \frac{P(have\ children|married) \cdot P(married)}{P(have\ children|married) \cdot P(married) + P(have\ children|commited\ relationship) \cdot P(commited\ relationship) + \ldots}$$

Bayesian Thinking

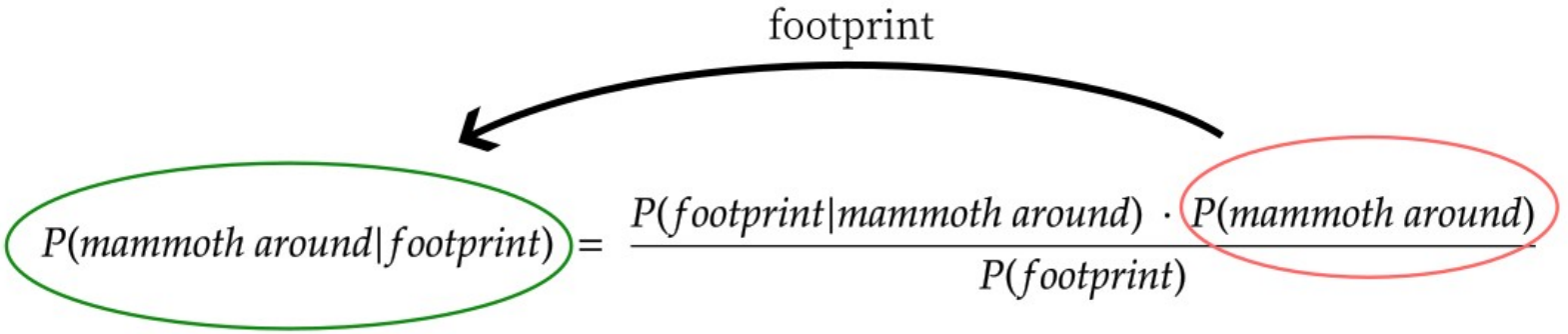
- Remember your **priors**
- Consider the strength of the **evidence**
- Don‘t discount small evidence (**snowflakes**)

- It‘s a natural way of thinking!
- Example: A prehistoric hunter searching for a mammoth

Bayesian Thinking

$$P(hypothesis|evidence) = \frac{P(evidence|hypothesis) \cdot P(hypothesis)}{P(evidence)}$$

$$P(mammoth\ around|footprint) = \frac{P(footprint|mammoth\ around) \cdot P(mammoth\ around)}{P(footprint)}$$



Remember your priors

- The probability BEFORE the new evidence
- Frequently forgotten a.k.a base-rate neglect

- The hunter knows that this time of the year in this region it is very likely that mammoths can be encountered. He expects to see two per week. (i.e. about 30% chance of seeing one on a random day).

Strength of Evidence

- What is strong evidence?
- Happens often?
- Happens seldomly?
- Something else?

Strength of Evidence

Evidence	Seen frequently?	Happens when mammoths are around?	Happens when mammoths are not around?
Huge pile of dung	Yes (41%)	Yes (90%)	Rarely (20%)
***Blue-Flute* bird can be heard**	No (18%)	Yes (60%)	No (0.1%)
Broken branches along the path	Yes (60%)	Yes (65%)	Yes (58%)
Earthquake	No (0.001%)	Yes (0.001%)	Yes (0.001%)
Green beetles in dung	Yes (80%)	Yes (80%)	Yes (80%)
Red beetles in dung	Yes (80%)	No (23%)	Yes (99%)
Blue beetles in dung	Yes (80%)	Yes (85%)	Yes (78%)

Strength of Evidence

- Three days ago I started a new diet and today I wake up and have more energy than in weeks. Does that mean the diet is working?
- Imagine a universe in which the diet works. Would it be possible to see this result?
- Imagine a universe in which the diet doesn‘t do anything. Would it be possible to see this result?

Snowflakes of evidence

- An avalanche is made of snowflakes
- Many small updates with tiny pieces of evidence can yield large results. Don‘t ignore them.
- Example: Hunter is told repeatedly by tribe elders there are no mammoths in this region. He also fails to find piles of dung. He encounters a tribe that hasn‘t seen a mammoth in years.

Exercise

- You are in the tram and a person (of whichever gender + age you are into) smiles at you. What is the likelihood they find you attractive? Make up all the numbers as needed.
- You are in a café and the person (of whichever gender + age you are into) serving you smiles at you. What is the likelihood they find you attractive?
- What is the difference?

Simplified Poker

- Ante: 10c (cent)
- 1 card each
- Alternate who starts
- First player can bet 10c or check
  - If first player betted, second player can bet 10c or fold
  - Else if first player checked, second player can check or bet 10c
    - If second player betted, first player can bet or fold
- If one player folds, no cards are shown

Takeaways

- Remember the **prior** probability (and ignore new evidence while doing so)
  - The posterior is always a **shift** relative to the prior
- Consider the **strength** of the evidence. Imagine alternative universes where your hypothesis is wrong. Is it possible you would see this evidence?
- Don‘t ignore tiny evidence (**snowflakes**) but update a tiny bit


I'M A STRICT BAYESIAN.
WHAT, LIKE YOU USE BAYESIAN STATISTICS?
NO, I MEAN SOMETIMES I CHANGE MY OPINION, AND WHEN I DO I FEEL LIKE IT'S THE RESULT OF MATH.
THAT SOUNDS LIKE NORMAL REASONING PLUS UNWARRANTED CONFIDENCE.
I AM ADJUSTING MY PRIORS ABOUT YOU, SIR.
smbc-comics.com

Useful Links

- Cleodora Forecasting (cleodora.org)
- Julia Galef on YouTube
  - “A visual guide to Bayesian thinking”
  - “Is Bayesian thinking a sham?“
- https://arbital.com/p/bayes_rule_guide/
- https://rootclaim.com
- “The Theory That Would Not Die” (Book/YouTube)

Sources & License

- Playing cards (published under LGPLv2) http://svg-cards.sourceforge.net/
- SMBC comic https://www.smbc-comics.com/comic/bayesian
- Simplified Poker https://www.lesswrong.com/posts/i2M3vWPBqyefh3uow/simplified-poker . Even though it‘s not specified in the source, it probably refers to Kuhn Poker https://en.wikipedia.org/wiki/Kuhn_poker
- Pictures of Thomas Bayes (public domain), Pierre-Simon Laplace (public domain) and 10 cent coin (fair use) from Wikipedia